The Coefficient of Price-Related Bias: A Measure of Vertical Equity

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The concept of vertical equity, which relates to the relationship between assessment levels and value ranges, has long been an area of interest and concern to both assessors and taxpayers. The only recognized measure or gauge of vertical equity currently available to assessors is the price-related differential (PRD). The profession has grappled with the measure. On the one hand, it has been incorporated into IAAO standards and training materials. On the other hand, assessment professionals warn that it is biased, is subject to sampling error and outliers, and should be supplemented by statistical testing. Most importantly, unlike measures of central tendency or the coefficient of dispersion (COD), the PRD does not provide a meaningful gauge of what it purports to measure. What does a PRD of 1.05 really mean and how much worse is it than a PRD of 1.03?

This article introduces a supplemental measure of vertical equity termed the coefficient of price-related bias (PRB). Importantly, it measures the percentage relationship between property values and assessment ratios and indicates by what percentage assessment levels change whenever property values are doubled (or halved). It also addresses technical deficiencies in the PRD and quantifies the statistical significance of any indicated inequities.

Background and Prior Research

Assessing officials have long been concerned with what has become known as *vertical equity*, that is, equity in the assessment of low- and high-value properties. However, the first attempt to measure vertical equity appears to be the *Census of Governments* quinquennial ratio studies of metropolitan and selected small jurisdictions conducted from 1957 to 1982 (Denne 2011). The U.S. Census Bureau first introduced the PRD in its 1957 study and reported it in each of the five succeeding studies. The 1982 report describes it as follows:

> *To obtain some notion of any association, within a jurisdiction, between levels of assessed values and particular property sales price ranges, a price-related differential is calculated. This was done only for single-family residential property.... This measure results from dividing the mean of the assessment ratios involved by the aggregate assessment-sales price ratio for the same sales.* (U.S. Department of Commerce, U.S. Census Bureau 1984, XLIII)

The PRD, as the measure came to be known, was thus intended to provide only a rough measure of *any* association between assessment levels and price levels. Measures below 1.00 suggested assessment progressivity, in which low-value properties were under-assessed relative to high-value properties. Measures above 1.00 suggested assessment regressivity, in which high-value properties were under-assessed relative to low-value properties.

Some researchers recognized that the PRD contained a downward bias, because the mean ratio is biased to the high side. *Improving Real Property Assessment: A Reference Manual* (Almy, Gloudemans, and Denne 1978, 126–128), the first textbook on mass appraisal, and its successors explained and illustrated the bias (Eckert, Gloudemans, and Almy 1990, 528–529, 539–540; Gloudemans 1999, 230–231, 239–240). Because of this bias, a standard for the PRD should be centered somewhat above 1.00. *Improving Real Property Assessment* offered the following guidelines:

> *In judging whether or not a comparison of the mean and weighted mean suggests assessment regressivity or progressivity, the following rule of thumb may be employed. If sample size is at least 30 and the ratio of the mean to the weighted mean exceeds 1.10, this is a good indication that assessments are regressive. If sample size is at least 30 and the ratio of the mean to the weighted mean is less than 0.95, this is a good indication that assessments are progressive. Again, however, remember that such differences may be due either to a general pattern or simply to one or more outliers.* (Almy, Gloudemans, and Denne 1978, 128)

Although the first IAAO *Standard on Ratio Studies* (IAAO 1980) did not contain a discussion of vertical equity or mention the PRD, the section on "Definitions" included the term and, consistent with *Improving Real Property Assessment*, stated that,

> *PRDs less than 1.0 are held to indicate assessment progressivity, and PRDs greater than 1.0 are held to indicate assessment regressivity, although there is a bias in the measure such that PRD in the general range of 0.90 to 1.10 are inconclusive indicators of assessment regressivity.* (IAAO 1980, 14)

Mention of the PRD in *Improving Real Property Assessment* and the 1980 *Standard on Ratio Studies* led to its frequent inclusion in assessment ratio studies. As experience was gained and the awareness of vertical equity concepts spread, practitioners became dissatisfied with the 0.90 to 1.10 guideline, thinking it sometimes implicitly condoned unacceptable degrees of vertical inequity. At the same time, a growing literature offered various statistical tests for vertical equity, many of which are nicely summarized in Sirmans, Diskan, and Friday (1995). More recent contributions include De Cesare and Ruddock (1998), Jensen (2009), Moore and Myers (2010), and McMillen (2011).

The 1990 *Standard on Ratio Studies* (IAAO 1990, 24–25) set forth a standard for the PRD of 0.98 to 1.03, explaining that

> *The reason this range is not centered on 1.00 relates to an inherent upward bias in the arithmetic mean (numerator in the PRD) that does not equally affect the weighed mean (denominator in the PRD).*

The standard warned that

> *When samples are small, have high dispersion, or include properties with extreme value, the PRD may not provide an accurate indication of assessment regressivity or progressivity.*

It also added that

> *It is good practice to perform an appropriate statistical test for price-related biases before concluding that they exist…*

The 0.98 and 1.03 standards were carried over to the 1999 and 2007 IAAO standards on ratio studies (IAAO 1999 and 2007), as were the admonitions about the potential unreliability of the PRD and the need for appropriate testing. In fact, the 2007 standard added the warning that

> *The price-related differential (PRD) and weighted mean are sensitive to sales with high prices even if the ratios on higher priced sales do not appear unusual relative to other sales.* (IAAO 2007, 11)

Appropriate statistical tests for regressivity listed in the 1990, 1999, and 2007 standards are correlation or regression analysis and the nonparametric Spearman-Rank test. Their application to testing for vertical equity is explained in Almy, Gloudemans, and Denne (1978), Eckert, Gloudemans, and Almy (1990), and Gloudemans (1999).

The Need for a More Meaningful Measure

Obviously, assessment officials have been long concerned with the issue of price-related biases and have sought a meaningful measure of its incidence. Still, the only measure or gauge of price-related biases that has been put forth is the PRD, for which numbers higher than 1.00 indicate increasing regressivity and numbers lower than 1.00 indicate increasing progressivity. But, aside from caveats that the measure is biased and may not be reliable, what does a PRD of a given value, say 1.03, really mean? Or how bad is a PRD of 0.95, 1.05, or 1.15?

The problem is not that the PRD is meaningless. It is simply what its originators meant it to be—an easy-to-calculate measure that provides "some notion of any association" between assessment ratios and property value. While the PRD stands as a familiar first indicator of price-related bias, assessors would benefit from a more meaningful supplemental measure. Just as assessors use multiple measures of central tendency and dispersion, so too they could use additional measures of vertical equity.

While helpful in testing the null hypothesis of no price-related bias, tests indicate only whether it can be concluded that *some* regressivity or progressivity is measured. The bias may be egregious or it may be benign, as it is well known that even the smallest of differences or relationships can be found statistically significant if sample size is large enough.

Is there a measure that, like the COD, provides a meaningful gauge of vertical equity and, if possible, also lends itself to statistical testing if the gauge enters the warning zone?

The Coefficient of Price-Related Bias

The coefficient of price-related bias provides one such measure. It measures the percentage increase (decrease) in assessment ratios relative to the percentage increase in property values. For example, a coefficient of 0.022 indicates that assessment ratios increase by 2.2 percent whenever values double (increase by 100 percent). A coefficient of –0.055 indicates that assessment ratios fall by 5.5 percent when values double.

The coefficient is obtained by regressing percentage differences from the median assessment ratio on percentage differences from the median value. The regression coefficient, which can be termed the coefficient of price-related bias (PRB), quantifies the relationship between property values and assessment levels. The dependent variable in the analysis is

(ratio – median ratio) ÷ median ratio.

For example, if the median ratio is 0.92, the dependent variable corresponding to a ratio of 1.05 would be

0.13 ÷ 0.92 = 0.141.

The independent variable makes use of a property value proxy (abbreviated *value* below) computed as follows:

LN(*value*) ÷ 0.693,

where *value* = 0.50 × appraised value + 0.50 × sale price. If the appraisal level is not close to 1.00, appraised values can be divided by the median ratio to ensure that they receive equal weight with sale prices.

The property value proxy is computed as one-half of (time-adjusted) sale price plus one-half of appraised value to minimize statistical bias that would overstate the degree of regressivity (or understate the degree of progressivity). If sale prices were used as the value proxy, sale prices that occurred on the high side of market value would result in somewhat high estimates of value and somewhat low assessment ratios (since sale price is the denominator of the ratios). This would result in a false indicator of assessment regressivity. If a property sold on the low side of market value, the value proxy would be pulled downward and the assessment ratio upward, again creating a regressivity bias. This well-known *errors in variables* problem is the same phenomenon that creates an upward bias in the mean assessment ratio and PRD. If appraised values were substituted as the value proxy, just the opposite would occur, resulting in a bias toward progressivity. While averaging sale prices and appraised values is not a perfect solution, it significantly reduces the bias (to the extent that assessments and sale prices have the same randomness or stochasticity, the bias is reduced toward zero).

The use of logarithms converts the value proxy to percentages and division by 0.693 (the natural logarithm of 2) permits each doubling of value to be associated with an increment of 1, as illustrated in table 1.

Table 1. Illustration of independent variable for PRB regression

Value	LN(*Value*) ÷ .693
50,000	15.613
100,000	16.613
200,000	17.613
400,000	18.613
800,000	19.613

Besides capturing percentage changes, the use of logarithms has the advantage of minimizing the effect of any outliers. Notice, for example, the tighter distribution of the right-hand column in the table versus the left-hand column.

The PRB indicates the percentage by which assessment ratios change as *value* changes by one unit, that is, is doubled or halved. Thus a PRB of 0.016 would indicate, for example, that ratios increase by 1.6 percent whenever values double (mild progressivity) and a PRB of –0.065 would indicate that ratios fall by 6.5 percent whenever values double (regressivity).

The analysis also quantifies the statistical strength or significance of the relationship. A PRB coefficient not significantly different from zero indicates that the null hypothesis of no price-related bias cannot be rejected. PRB coefficients that differ from zero by meaningful amounts and that are statistically significant warrant attention. Price-related bias may be noted when (a) the PRB coefficient is less than –0.03 or greater than 0.03 and (b) the relationship is statistically significant at the 95 percent confidence level. Coefficients below –0.05 or greater than 0.05 could be viewed with concern, again assuming they are significant at the 95 percent confidence level.

Calculation Example

Figures 1 and 2 and table 2 present an example of the PRB calculation for a sample of 30 sales with a median assessment-sales ratio (ASR) of 0.841, COD of .087, and PRD of 1.030. Because the median is meaningfully different from 1.00, the value proxy is computed as 0.50 of sale price (SP) + 0.50 of appraised value (AV) divided by the median ratio:

Value = 0.50 × SP + 0.50 × (AV ÷ Median)

The independent variable in the analysis is the natural logarithm of the value proxy divided by 0.693. The dependent variable is (ASR – Median) ÷ Median.

Regression of (ASR – Median) ÷ Median on LN(*Value*) ÷ 0.693 produces a PRB coefficient of –0.136, indicating that ratios decrease by 13.6 percent each time value doubles. The *t*-value of –4.476 and significance value of .000

Table 2. Example of PRB calculation

(1)	(2)	(3)	(4)	(5)	.5(3) + .5(5)	Indep Var	Dep Variable
							(ASR–Med)/
Sale	AV	SP	ASR	AV/Med	*Value*	LN(*Value*)/.693	Med
1	131,670	139,500	0.944	156,522	148,011	17.179	0.122
2	170,190	175,950	0.967	202,313	189,131	17.533	0.150
3	152,820	155,000	0.986	181,664	168,332	17.365	0.172
4	152,370	169,700	0.898	181,129	175,415	17.424	0.067
5	164,340	159,250	1.032	195,359	177,304	17.440	0.227
6	156,870	182,900	0.858	186,479	184,689	17.498	0.020
7	167,670	205,000	0.818	199,317	202,159	17.629	–0.028
8	179,010	230,350	0.777	212,797	221,574	17.761	–0.076
9	183,600	215,650	0.851	218,254	216,952	17.731	0.012
10	175,590	199,950	0.878	208,732	204,341	17.644	0.044
11	187,200	240,000	0.780	222,533	231,267	17.823	–0.073
12	197,820	224,900	0.880	235,158	230,029	17.815	0.046
13	207,180	275,000	0.753	246,284	260,642	17.996	–0.104
14	198,990	289,000	0.689	236,549	262,774	18.007	–0.181
15	223,830	260,850	0.858	266,077	263,464	18.011	0.020
16	216,630	290,000	0.747	257,518	273,759	18.066	–0.112
17	241,560	294,000	0.822	287,153	290,577	18.152	–0.023
18	238,950	249,500	0.958	284,051	266,775	18.029	0.138
19	247,410	329,000	0.752	294,108	311,554	18.253	–0.106
20	240,300	279,000	0.861	285,656	282,328	18.111	0.024
21	283,680	335,000	0.847	337,223	336,112	18.362	0.007
22	285,120	314,000	0.908	338,935	326,468	18.320	0.079
23	310,950	397,500	0.782	369,641	383,570	18.553	–0.070
24	302,220	389,000	0.777	359,263	374,131	18.517	–0.076
25	318,600	369,000	0.863	378,734	373,867	18.516	0.026
26	341,010	459,000	0.743	405,374	432,187	18.725	–0.117
27	357,570	427,900	0.836	425,060	426,480	18.706	–0.007
28	354,240	526,000	0.673	421,101	473,551	18.857	–0.199
29	411,930	545,500	0.755	489,680	517,590	18.985	–0.102
30	415,440	590,000	0.704	493,853	541,926	19.052	–0.163
Sum	7,214,760	8,917,400	24.997			PRB	–0.136
						STD Error	0.030
	Median	0.841		COD	0.087	t–value	–4.476
	Mean	0.833		PRD	1.030	d.f.	28
	WtdMean	0.809		Sales	30	Sig	0.000

confirm the statistical significance of the relationship.

Figures 1 and 2 illustrate the relationships. Figure 1 plots assessment ratios against the value proxy; as value increases, ratios fall. Figure 2 converts the relationship to percentages. Each increase of 1.00 on the horizontal axis (e.g., from 17.50 to 18.50) represents a doubling of value. When values double, the vertical axis falls from values generally above zero to values generally below zero. Although figure 1 is all that is needed to visualize the relationship between value and ratios, figure 2 is presented to illustrate calculation of the PRB coefficient, which represents the slope of the regression when fit to the data points (in this case, –0.136).

PRB analyses can be produced with a spreadsheet program, as illustrated in table 2, or with a statistical package like SPSS. Figure 3 contains SPSS syntax for computing traditional ratio statistics, the PRB, and a graph of ratios with the value proxy for the 30 sales shown in table 2 (aside from comments, the program is only 10 lines). The regression command produces the output shown in table 3.

Table 3. Output from PRB regression

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.646	.417	.396	.08329

Coefficients

	Model	Unstandardized Coefficients B	Unstandardized Coefficients Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	2.446	.549		4.457	.000
	LN_VALUE	–.136	.030	–.646	–4.476	.000

Empirical Results

PRB analyses were conducted for a very large assessment agency that had completed a revaluation of both urban and sparsely populated rural areas. The revaluation covered 66 residential market areas with sample sizes ranging from less than 100 to more than 2,500 time-adjusted sales. Sixteen of the 66 market areas (24 percent) contained PRB coefficients outside of –0.05 to 0.05 that were significant at the 95 percent confidence level. Five were less than –0.10 or greater than 0.10. All other areas had PRB coefficients that were between –0.05 and 0.05 or were insignificant at the 95 percent confidence level.

Figure 1. PRB plot of assessment ratios against value proxy

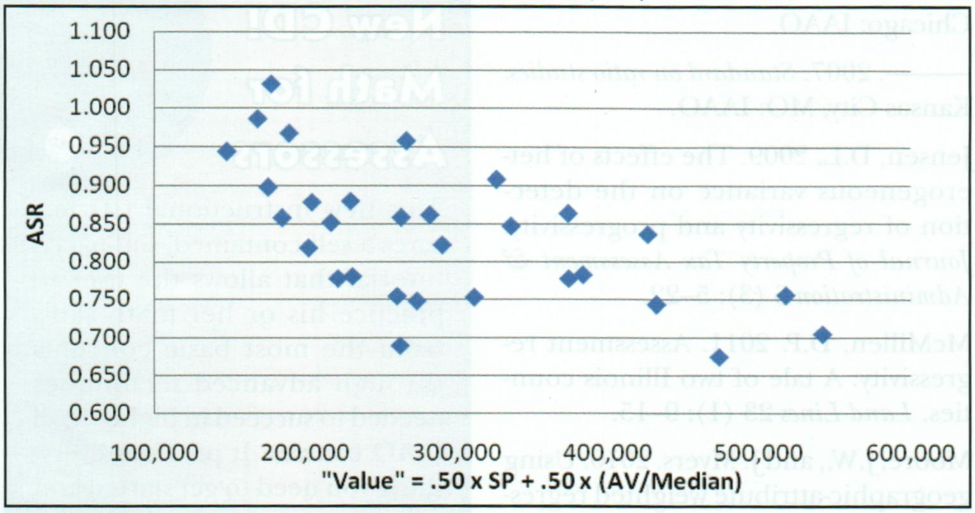

Figure 2. PRB plot of assessment ratios against value proxy as percentages

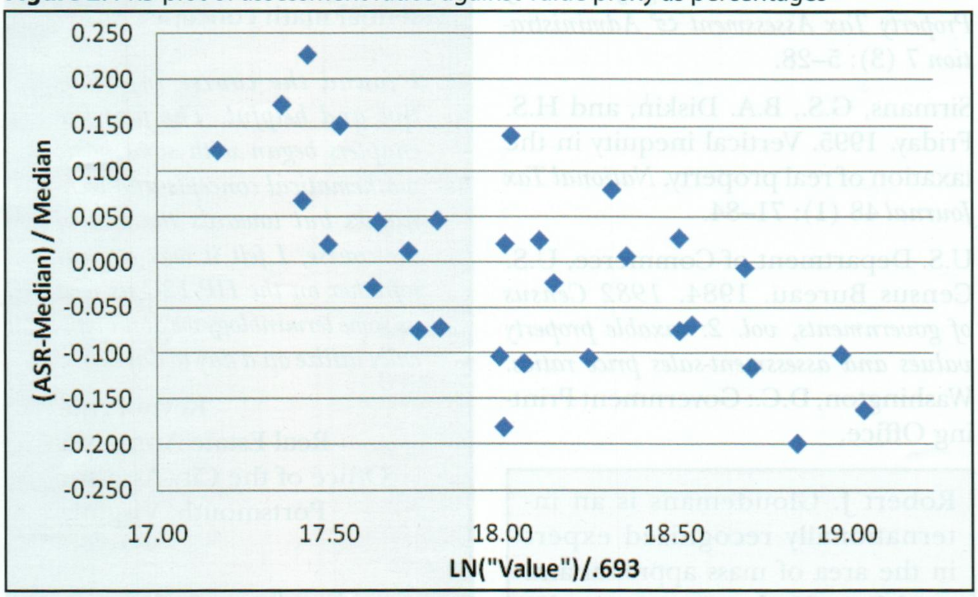

Figure 3. SPSS syntax for computing PRB

```
*RATIO STATISTICS.
RATIO STATS AV WITH SP/PRINT=MEDIAN MEAN WGTMEAN MIN MAX PRD COD.
*SAVING MEDIAN RATIO.
COMPUTE DATASET=1.
COMPUTE RATIO=AV/SP.
AGGREGATE OUTFILE='C:\TEMP\TEMP.SAV'/BREAK=DATASET/MEDIAN_RATIO=MEDIAN(RATIO).
MATCH FILES FILE=*/TABLE='C:\TEMP\TEMP.SAV'/BY=DATASET.
*DEPENDENT VARIABLE - PCT DIFFERENCE FROM MEDIAN RATIO.
COMPUTE PCT_DIFF=(RATIO-MEDIAN_RATIO)/MEDIAN_RATIO.
*INDEPENDENT VARIABLE - LN(VALUE/MED_VALUE)/.693.
COMPUTE VALUE=.50*(AV/MEDIAN_RATIO)+.50*SP.
COMPUTE LN_VALUE=LN(VALUE)/.693.
GRAPH/SCATTER=VALUE WITH RATIO.
REGRESSION/DEP=PCT_DIFF/ENTER=LN_VALUE.
```

Interestingly, most market areas that were significant at the 95 percent confidence level had coefficients that fell between −0.05 and 0.05. In fact, a number fell within a tight range of −0.03 to 0.03, illustrating that statistical significance does not necessarily imply cause for concern.

The primary advantage of the PRB is that it provides a meaningful gauge of vertical equity, as well as the statistical strength of any relationship. Assessment administrators can then address any indicated problem areas with confidence in the meaningfulness of results and need for further investigation or action.

Finally, note that, like the PRD, the PRB can be affected by *heteroscedasticity*, that is, an increase in the dispersion of the dependent variable (ratios) over the range of the independent variable (value); see Jensen (2011). Serious cases of heteroscedasticity, which can skew results, can be noted by plotting ratios against value as part of the PRB analysis.

Summary and Conclusions

For many years the PRD has provided the only gauge of vertical bias available to assessors. While easy to calculate, the PRD suffers in a number of respects, the most important being that it lacks intuitive appeal and meaningfulness. It is also suffers measurement bias, can be skewed by outliers, and does not lend itself to statistical testing.

Although the PRD will undoubtedly continue to provide a familiar, first-line indicator of vertical bias, the PRB coefficient provides a more meaningful and reliable measure that can either supplement or be used in place of the PRD. Ideally, a measure of vertical equity would quantify the relationship between percentage changes in value and assessment ratios. The PRB coefficient does so. It also addresses the bias problem, minimizes the effect of outliers, and measures the statistical significance of any indicated inequities.

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